Efficient Nash Equilibrium Computation through Monte Carlo Counterfactual Regret Minimization

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Counterfactual Regret Minimization (CFR)

- In two-player zero-sum games, **Nash equilibrium** strategies (minimax strategies) are **unexploitable**: they will do no worse than tie on expectation against any opponent.
- **CFR** is a state-of-the-art iterative algorithm for approximating Nash equilibria in two-player zero-sum games. It resembles self-play over a series of T games.
- By **minimizing regret** (improving the strategy) at each decision point independently, the entire strategy converges towards a Nash equilibrium.
- CFR is **memory efficient**, **straightforward to implement**, and **easy to optimize and parallelize**.
- Monte-Carlo CFR is a family of **sampling variants** that converge much faster in practice than the base algorithm. This paper proposes **Public Chance Sampling** and shows that it converges faster than earlier approaches.

"Vanilla" CFR, 2007
In each iteration, enumerate all chance events and update the complete game tree. Not useful in practice.

↓ Very fast but noisy iterations.

Chance Sampling (CS), 2007
O(1) terminal node evaluation
[2-4] speed: 1.25m iter/sec
Sample one event for us
Update our strategy considering one opponent private chance event.

→ Slower iterations, Lower variance

Self / Public Chance Sampling (SPCS)
O(n) terminal node evaluation
[2-4] speed: 1952 iter/sec
Sample one event for us, but update while considering all *n* opponent private chance events.

↓ Slower iterations, More updates per iteration (from CS)

↓ Same time complexity, More updates per iteration (from SPCS)

Opponent / Public Chance Sampling (OPCS)
O(n) terminal node evaluation
[2-4] speed: 1414 iter/sec
Update all *n* of our chance events with respect to one sampled opponent event.

→ Same time complexity, Lower variance

Public Chance Sampling (PCS)
Possible O(n) terminal node evaluation
[2-4] speed: 709 iter/sec
Sample public chance events, but consider all *n* private events for each player

Algorithm outline:

Initialize two strategies and repeatedly traverse the game tree. This resembles a self-play algorithm.

At each decision *I*, use recursion to get the **value** of each action *a* and accumulate **regret**:

$$R_i^T(I,a) = \frac{1}{T}\sum_{t=1}^{T} \pi_{-i}^{\sigma^t}(I)\left(u_i(\sigma^t|_{I\rightarrow a}, I) - u_i(\sigma^t, I)\right)$$

Update the strategies proportional to their accumulated positive regret:

$$\sigma_i^{T+1}(I,a) = \frac{R_i^{T,+}(I,a)}{\sum_{a\in A(I)} R_i^{T,+}(I,a)}$$

Following this procedure, the **average strategy** used by the players converges to a Nash equilibrium.

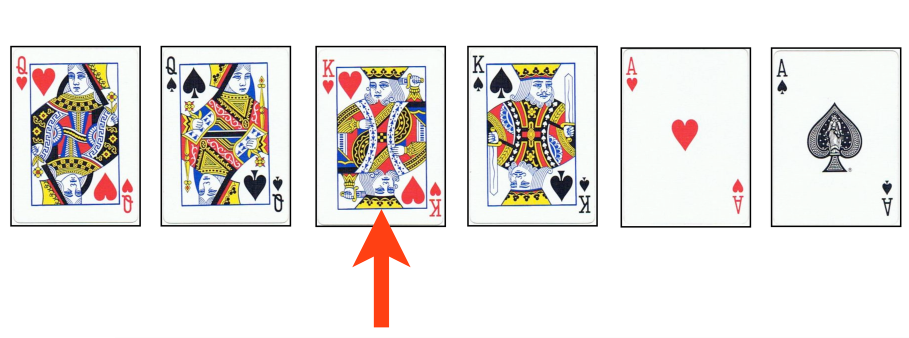
Sampling some or all of the chance events lets us perform fast, noisy updates, and this converges faster. **We can trade off between**:

- Iteration Speed
- Strategy updates per iteration
- Accuracy in estimating action values

Fast Terminal Node Evaluation (IJCAI 2011)

By exploiting game structure, a fast O(*n*) terminal evaluation may be possible when comparing *n* private states for each player.

This allows PCS to do the work of both OPCS and SPCS with the same time complexity!



Obvious O(n²) algorithm:

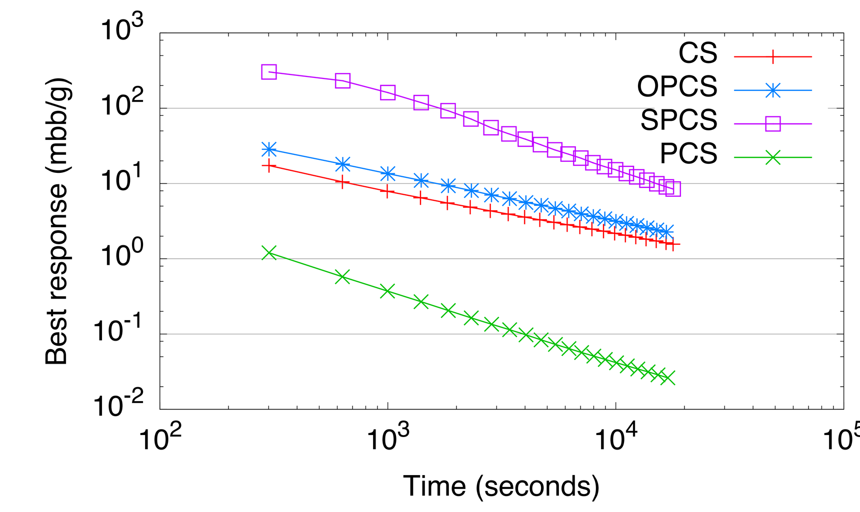
```
for( each of my hands x )
  for( each of their hands y )
    if( x > y )
      util[x] += payoff * P(y)
    else if( x < y )
      util[x] -= payoff * P(y)
```

Faster O(n) algorithm:

```
p_lose = total_prob;  p_win = 0;
for( each hand x )  //red arrow
above
  p_lose -= prob[x]
  util[x] = (p_win - p_lose)*payoff
  p_win += prob[x]
```

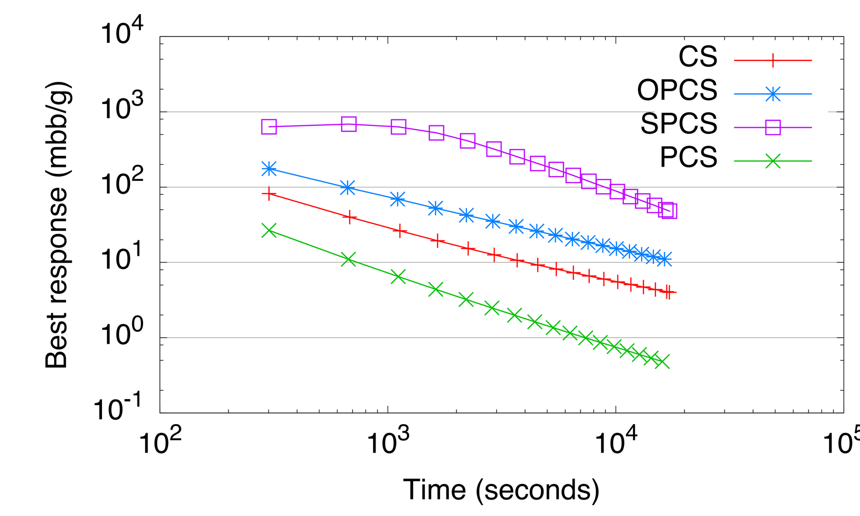
[2-Round, 1-Bet] Hold'em
A small poker game where strategies can be quickly created and evaluated.

- Y-axis shows distance to Nash equilibrium
- Game has 16 million information sets
- First PCS datapoint has already converged closer than final CS datapoint!



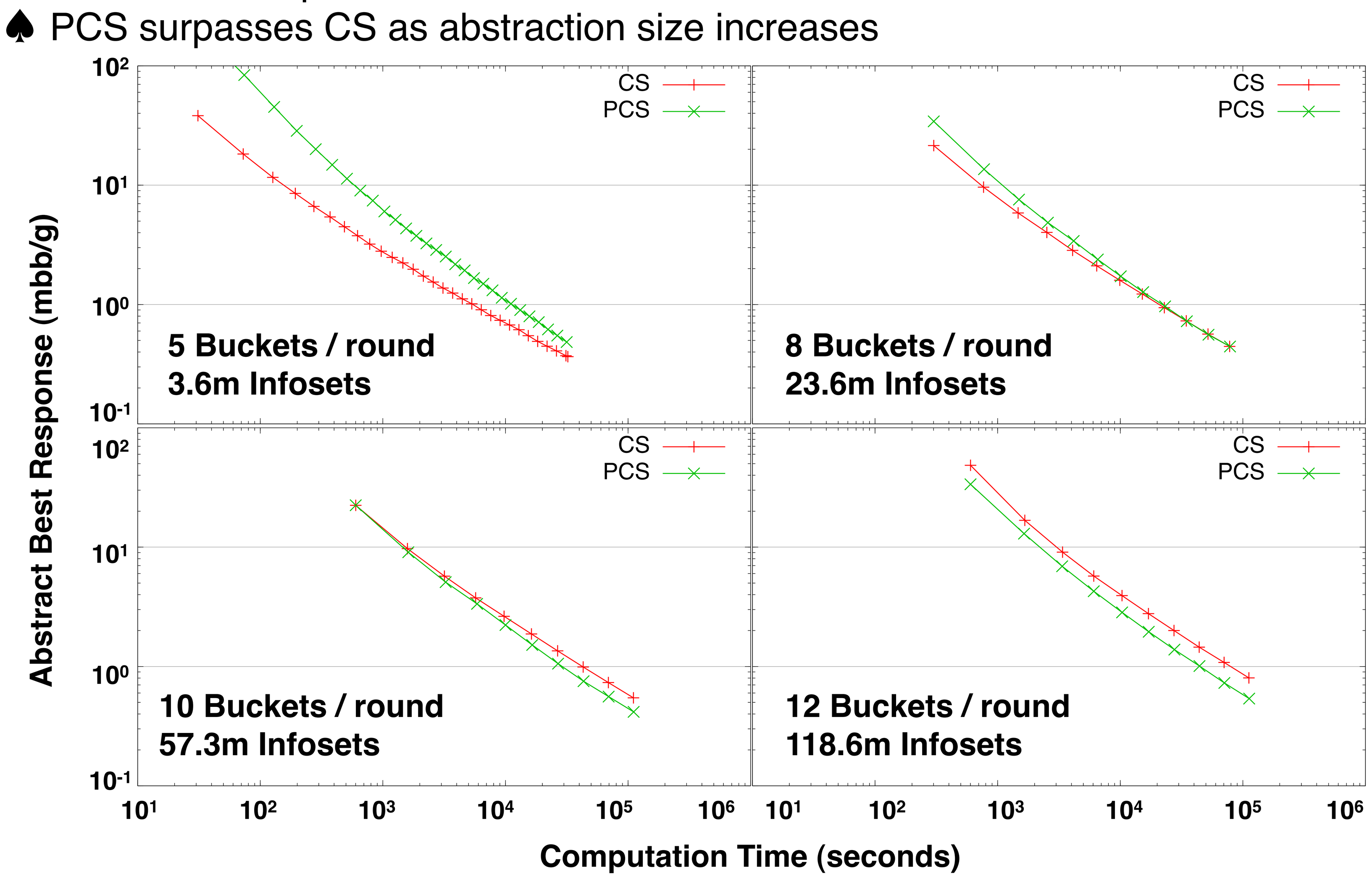
[2-Round, 4-Bet] Hold'em
A larger test domain that increases the players' action space

- 94 million information sets
- PCS curve is both lower and has a steeper slope



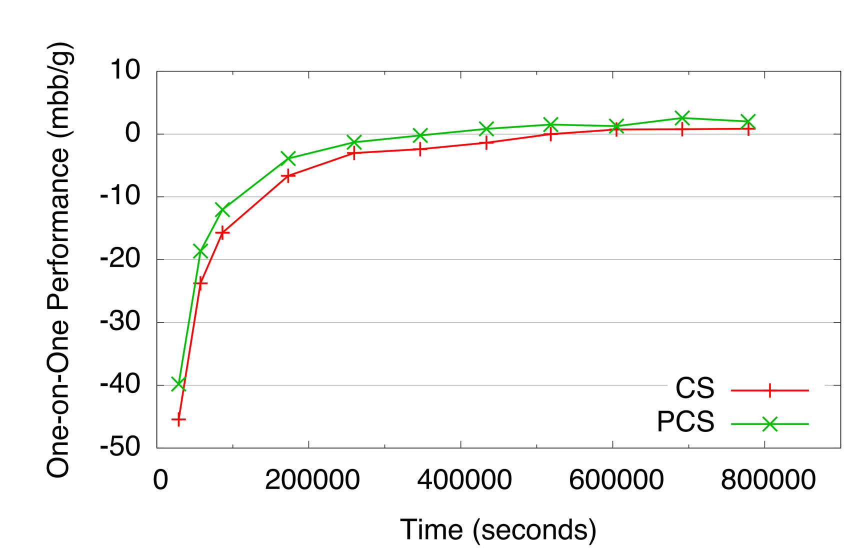
Limit Texas Hold'em: Abstract Best Response

- Real game: 10^14 information sets. Abstraction lets us produce tractable games.
- Increasing abstraction granularity results in better real game strategies, but increases computational costs
- PCS surpasses CS as abstraction size increases



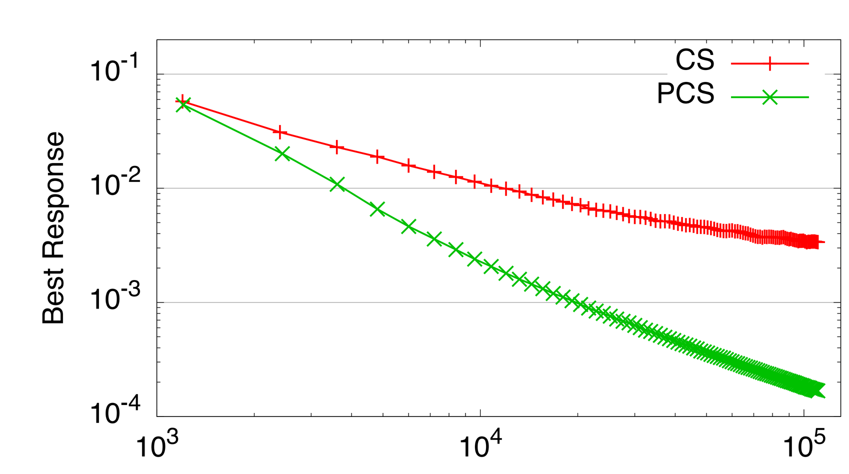
Limit Texas Hold'em: In-Game Performance

- In large abstractions, we can evaluate by in-game performance against a strong opponent (Hyperborean2011)
- Note the horizontal distance. CS must be run for much longer to reach the same level of performance.
- Abstraction has 880m information sets.



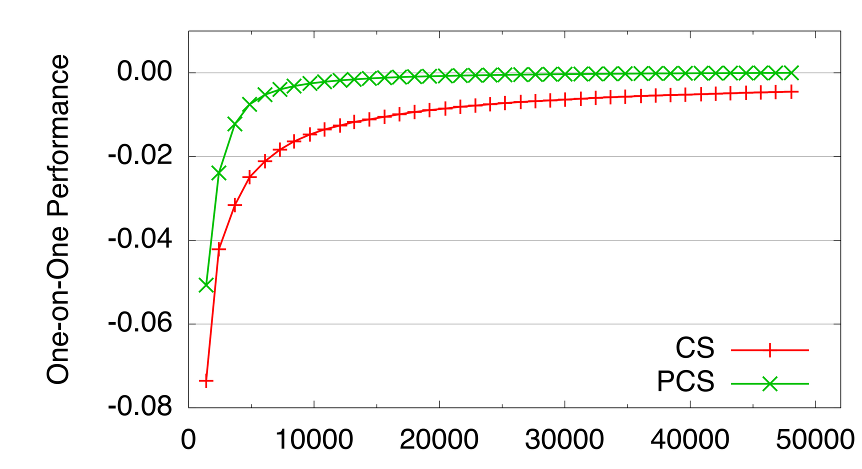
(2,2) Bluff: Exploitability

- Bluff is a 2-player dice game. Each player secretly rolls 2 dice and players bid on how many of each side was rolled.
- No public chance events, so PCS does efficient complete traversals.
- PCS' curve is both lower and steeper at each timestep.



(2,2) Bluff: In-Game Performance

- In this graph, we use the PCS and CS strategies to play against the final PCS strategy.
- PCS generates strong strategies much more quickly than CS. Consider the horizontal distance.



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